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U.2434 In triangle ABC the following relationship holds:

$$e^a+e^b+e^c+2\sqrt{e^{a+b}}+2\sqrt{e^{b+c}}+2\sqrt{e^{c+a}}\geq 9(e^{2s})^{\frac{1}{3}}$$

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Solution by Titu Zvonaru-Romania

Applying $AM-GM$ inequality, it follows that

$$e^c+\sqrt{e^{a+b}}+\sqrt{e^{a+b}}\geq 3\left(e^{c+\frac{a+b}{2}+\frac{a+b}{2}}\right)^{\frac{1}{3}}=3(e^{2s})^{\frac{1}{3}}$$

$$e^a+\sqrt{e^{b+c}}+\sqrt{e^{b+c}}\geq 3\left(e^{a+\frac{b+c}{2}+\frac{b+c}{2}}\right)^{\frac{1}{3}}=3(e^{2s})^{\frac{1}{3}}$$

$$e^b+\sqrt{e^{c+a}}+\sqrt{e^{c+a}}\geq 3\left(e^{b+\frac{c+a}{2}+\frac{c+a}{2}}\right)^{\frac{1}{3}}=3(e^{2s})^{\frac{1}{3}}.$$

Adding these inequalities, we obtain the desired inequality.

Equality holds if and only if $a=b=c$.