U.2472 If $t,u\geq 0$ and $x,y,z>0$, then in triangle ABC holds:

$$\frac{x+y}{z}\cdot a^t b^u+\frac{y+z}{x}\cdot b^t c^u+\frac{z+x}{y}\cdot c^t a^u\geq 2^{1+t+u}(3)^{\frac{4-t-u}{4}}\left(\sqrt{F}\right)^{t+u}$$

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Applying $AM-GM$ inequality, Cesaro's inequality $(x+y)(y+z)(z+x)\geq 8xyz$

and Carlitz's inequality $(abc)^{2/3}\geq \frac{4}{\sqrt{3}}F$, it follows that

$$\frac{x+y}{z}\cdot a^t b^u+\frac{y+z}{x}\cdot b^t c^u+\frac{z+x}{y}\cdot c^t a^u\geq$$

$$\geq 3\left(\frac{x+y}{z}\cdot a^t b^u\cdot\frac{y+z}{x}\cdot b^t c^u\cdot\frac{z+x}{y}\cdot c^t a^u\right)^{\frac{1}{3}}\geq$$

$$\geq 6(abc)^{\frac{t+u}{3}}\geq 6\left(\frac{4}{\sqrt{3}}F\right)^{\frac{t+u}{3}}=2^{1+t+u}(3)^{\frac{4-t-u}{4}}\left(\sqrt{F}\right)^{t+u}.$$

Equality holds if and only if triangle ABC is equilateral and $x=y=z$.