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U.2526 If $a,b,c\in R_+^*, m\in R_+$, and $a+b+c=1$, then

$$\frac{a^{m+2}}{(a+b)^m}+\frac{b^{m+2}}{(b+c)^m}+\frac{c^{m+2}}{(c+a)^m}\geq\frac{1}{3\cdot 2^m}$$

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Applying Radon's inequality and the known inequalities:

$$ab+bc+ca\leq a^2+b^2+c^2, 3(a^2+b^2+c^2)\geq(a+b+c)^2,$$

it follows that

$$\frac{a^{m+2}}{(a+b)^m}+\frac{b^{m+2}}{(b+c)^m}+\frac{c^{m+2}}{(c+a)^m}=\frac{a^{2m+2}}{(a^2+ab)^m}+\frac{b^{2m+2}}{(b^2+bc)^m}+\frac{c^{2m+2}}{(c^2+ca)^m}=$$

$$=\frac{(a^2)^{m+1}}{(a^2+ab)^m}+\frac{(b^2)^{m+1}}{(b^2+bc)^m}+\frac{(c^2)^{m+1}}{(c^2+ca)^m}\geq\frac{(a^2+b^2+c^2)^{m+1}}{(a^2+b^2+c^2+ab+bc+ca)^m}\geq$$

$$=\frac{(a^2+b^2+c^2)^{m+1}}{(a^2+b^2+c^2+a^2+b^2+c^2)^m}=\frac{(a^2+b^2+c^2)^{m+1}}{2^m(a^2+b^2+c^2)^m}=$$

$$=\frac{a^2+b^2+c^2}{2^m}\geq\frac{(a+b+c)^2}{3\cdot 2^m}=\frac{1}{3\cdot 2^m}.$$

Equality holds if and only if $a=b=c=\frac{1}{3}$.