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U.2527 If $a,b,c\in R_+^*, m\in R_+, x,y\in R_+^*$ and $a+b+c=1$, then

$$\frac{a^{m+2}}{(xa+yb)^m}+\frac{b^{m+2}}{(xb+yc)^m}+\frac{c^{m+2}}{(xc+ya)^m}\geq\frac{1}{3(x+y)^m}$$

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Applying Radon's inequality and the known inequalities

$ab+bc+ca\leq a^2+b^2+c^2, 3(a^2+b^2+c^2)\geq(a+b+c)^2$, it follows that

$$\frac{a^{m+2}}{(xa+yb)^m}+\frac{b^{m+2}}{(xb+yc)^m}+\frac{c^{m+2}}{(xc+ya)^m}=$$

$$=\frac{a^{2m+2}}{(xa^2+yab)^m}+\frac{b^{2m+2}}{(xb^2+ybc)^m}+\frac{c^{2m+2}}{(xc^2+yca)^m}=$$

$$=\frac{(a^2)^{m+1}}{(xa^2+yab)^m}+\frac{(b^2)^{m+1}}{(xb^2+ybc)^m}+\frac{(c^2)^{m+1}}{(xc^2+yca)^m}\geq$$

$$\geq\frac{(a^2+b^2+c^2)^{m+1}}{(x(a^2+b^2+c^2)+y(ab+bc+ca))^m}\geq$$

$$=\frac{(a^2+b^2+c^2)^{m+1}}{\left(x(a^2+b^2+c^2)+y(a^2+b^2+c^2)\right)^m}=\frac{(a^2+b^2+c^2)^{m+1}}{(x+y)^m(a^2+b^2+c^2)^m}=$$

$$=\frac{a^2+b^2+c^2}{(x+y)^m}\geq\frac{(a+b+c)^2}{3(x+y)^m}=\frac{1}{3(x+y)^m}.$$

Equality holds if and only if $a=b=c=\frac{1}{3}$.