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U.2551 If $x, y, z > 0, t \geq 0$, then in any triangle ABC holds

$$\frac{x+y+2t}{z+t}\cdot a+\frac{y+z+2t}{x+t}\cdot b+\frac{z+x+2t}{y+t}\cdot c\geq 4(3)^{1/4}\sqrt{F}$$

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Denoting $m=x+t, n=y+t, q=z+t$, the given inequality becomes:

$$\frac{m+n}{q}\cdot a+\frac{n+q}{m}\cdot b+\frac{q+m}{n}\cdot c\geq 4(3)^{\frac{1}{4}}\sqrt{F}\quad (1)$$

Applying $AM-GM$ inequality, Cesaro's inequality $(m+n)(n+q)(q+m)\geq 8mnq$

and Carlitz's inequality $(abc)^{2/3}\geq \frac{4}{3}\sqrt{3}F$, it follows that:

$$\frac{m+n}{q}\cdot a+\frac{n+q}{m}\cdot b+\frac{q+m}{n}\cdot c\geq 3\left(\frac{m+n}{q}\cdot\frac{n+q}{m}\cdot\frac{q+m}{n}\right)^{\frac{1}{3}}(abc)^{\frac{1}{3}}\geq$$

$$\geq 6\left(\frac{4}{3}\sqrt{3}F\right)^{\frac{1}{2}}=4\left(\sqrt{3}\right)^{\frac{1}{4}}\sqrt{F}$$

Equality holds if and only if $a=b=c, m=n=q$, that is $x=y=z$.