

ROMANIAN MATHEMATICAL MAGAZINE

U.2578 If $t \geq 0, x, y > 0$, and in the triangle ABC , $m_a + m_b + m_c = 2m$ such that $mx > y \max\{m_a, m_b, m_c\}$, then holds:

$$\frac{m_a^{t+2}}{(xm - ym_a)^t} + \frac{m_b^{t+2}}{(xm - ym_b)^t} + \frac{m_c^{t+2}}{(xm - ym_c)^t} \geq \frac{3\sqrt{3} \cdot 2^t}{(3x - 2y)^t} \cdot F$$

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Applying Radon's inequality, the known inequality $xy + yz + zx \leq x^2 + y^2 + z^2$, the formula $m_a^2 + m_b^2 + m_c^2 = \frac{3(a^2+b^2+c^2)}{4}$ and Ionescu-Weitzenbock's inequality

$a^2 + b^2 + c^2 \geq 4\sqrt{3}F$, it follows that:

$$\begin{aligned} & \frac{m_a^{t+2}}{(xm - ym_a)^t} + \frac{m_b^{t+2}}{(xm - ym_b)^t} + \frac{m_c^{t+2}}{(xm - ym_c)^t} = \\ &= \frac{m_a^{2t+2}}{(xmm_a - ym_a^2)^t} + \frac{m_b^{2t+2}}{(xmm_b - ym_b^2)^t} + \frac{m_c^{2t+2}}{(xmm_c - ym_c^2)^t} = \\ &= \frac{(m_a^2)^{t+1}}{(xmm_a - ym_a^2)^t} + \frac{(m_b^2)^{t+1}}{(xmm_b - ym_b^2)^t} + \frac{(m_c^2)^{t+1}}{(xmm_c - ym_c^2)^t} \geq \\ &\geq \frac{(m_a^2 + m_b^2 + m_c^2)^{t+1}}{\left(xm(m_a + m_b + m_c) - y(m_a^2 + m_b^2 + m_c^2)\right)^t} = \\ &= \frac{(m_a^2 + m_b^2 + m_c^2)^{t+1} \cdot 2^t}{\left((x - 2y)(m_a^2 + m_b^2 + m_c^2) + 2x(m_a m_b + m_b m_c + m_c m_a)\right)^t} = \\ &\geq \frac{(m_a^2 + m_b^2 + m_c^2)^{t+1} \cdot 2^t}{\left((x - 2y)(m_a^2 + m_b^2 + m_c^2) + 2x(m_a^2 + m_b^2 + m_c^2)\right)^t} = \frac{(m_a^2 + m_b^2 + m_c^2) \cdot 2^t}{(3x - 2y)^t} = \\ &= \frac{3(a^2 + b^2 + c^2) \cdot 2^t}{4(3x - 2y)^t} \geq \frac{12\sqrt{3}F \cdot 2^t}{4(3x - 2y)^t} = \frac{3\sqrt{3} \cdot 2^t}{(3x - 2y)^t} \cdot F \end{aligned}$$

Equality holds if and only if the triangle ABC is equilateral.