If $x, y, z > 0$, $x + y + z = 3$ then:

$$\frac{x^5}{x^2 + 1} + \frac{y^5}{y^2 + 1} + \frac{z^5}{z^2 + 1} \geq \frac{3}{2}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Tapas Das-India

$Lemma : \dfrac{t^5}{t^2 + 1} \geq 2t - \dfrac{3}{2}, t \in (0, 3)$

$Proof: \dfrac{t^5}{t^2 + 1} \geq 2t - \dfrac{3}{2} \ or \ 2t^5 - 4t^3 + 3t^2 - 4t + 3 \geq 0$

$$(t - 1)^2 (2t^3 + 4t^2 + 2t + 3) \geq 0 \ true \ as \ t < 3$$

$$\frac{x^5}{x^2 + 1} + \frac{y^5}{y^2 + 1} + \frac{z^5}{z^2 + 1} = \sum \frac{x^5}{x^2 + 1} \overset{Lemma}{\geq}$$

$$\geq \sum \left(2x - \frac{3}{2}\right) = 2(x + y + z) - \frac{9}{2} = 2 \times 3 - \frac{9}{2} = \frac{3}{2}$$

$$Equality \ holds \ for \ x = y = z = 1$$