ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\lambda\ge 0$ then:

$$\sum\frac{a^4+\lambda b^4}{ab}+(\lambda+1)\sum ab\ge 2(\lambda+1)\sum a^2$$

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Solution by Tapas Das-India

$$\sum\frac{a^4+\lambda b^4}{ab}=\sum\frac{a^4}{ab}+\lambda\sum\frac{b^4}{ab}=\sum\frac{(a^2)^2}{ab}+\lambda\sum\frac{(b^2)^2}{ab}$$

$$\overset{CBS}{\ge}\frac{(\sum a^2)^2}{\sum ab}+\lambda\frac{(\sum a^2)^2}{\sum ab}=(\lambda+1)\frac{(\sum a^2)^2}{\sum ab}$$

$$\textit{We need to show}:$$

$$\sum\frac{a^4+\lambda b^4}{ab}+(\lambda+1)\sum ab\ge 2(\lambda+1)\sum a^2$$

$$(\lambda+1)\frac{(\sum a^2)^2}{\sum ab}+(\lambda+1)\sum ab\ge 2(\lambda+1)\sum a^2$$

$$\frac{(\sum a^2)^2}{\sum ab}+\sum ab\ge 2\sum a^2$$

$$x+\frac{1}{x}\overset{\frac{\sum a^2}{\sum ab}=x\ge 1}{\ge} 2\ or\ (x-1)^2\ge 0\ true$$

Equality case for $a=b=c$.