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If $a,b,c>0,\lambda\geq 0$ then:

$$\sum\frac{(a+\lambda b)(a+\lambda c)}{bc}\geq 3(\lambda+1)^2$$

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Solution by Tapas Das-India

$$\sum\frac{(a+\lambda b)(a+\lambda c)}{bc}=\sum\left(\left(\frac{a}{b}+\lambda\right)\left(\frac{a}{c}+\lambda\right)\right)\overset{C-S}{\geq}\sum\left(\frac{a}{\sqrt{bc}}+\lambda\right)^2\overset{CBS}{\geq}$$

$$\geq\frac{1}{3}\left(\frac{a}{\sqrt{bc}}+\frac{b}{\sqrt{ca}}+\frac{c}{\sqrt{ab}}+3\lambda\right)^2\overset{AM-GM}{\geq}\frac{1}{3}(3+3\lambda)^2=3(\lambda+1)^2$$

Equality holds for $a=b=c$.