ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0$ then:

$$8\left(\frac{a^2}{b+1}+\frac{b^2}{a+1}\right)+3\left(\frac{1}{a^2}+\frac{1}{b^2}\right)\geq 14$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$8\left(\frac{a^2}{b+1}+\frac{b^2}{a+1}\right)+3\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=$$

$$=8\left(\frac{a^2}{b+1}+\frac{b^2}{a+1}\right)+3\left(\frac{1^3}{a^2}+\frac{1^3}{b^2}\right)\overset{Bergstrom\ \&\ Radon}{\geq}$$

$$\geq\frac{8(a+b)^2}{a+b+2}+\frac{3(1+1)^3}{(a+b)^2}\overset{a+b=t>0}{=}\frac{8t^2}{t+2}+\frac{24}{t^2}$$

$$We\ need\ to\ show\ \frac{8t^2}{t+2}+\frac{24}{t^2}\geq 14\ or$$

$$8t^4+24t+48\geq 14t^3+28t^2\ or$$

$$8t^4-14t^3-28t^2+24t+48\geq 0$$

$$or\ (t-2)^2(8t^2+18t+12)\geq 0\ true$$

$$Equality\ holds\ for\ t=a+b=2\ or\ \ a=b=1$$