ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$4\left(\frac{a^3}{b+1}+\frac{b^3}{a+1}\right)+5\left(\frac{1}{a}+\frac{1}{b}\right)\geq 14$$

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Solution by Tapas Das-India

$$4\left(\frac{a^3}{b+1}+\frac{b^3}{a+1}\right)+5\left(\frac{1}{a}+\frac{1}{b}\right)=$$

$$=\ 4\left(\frac{a^3}{b+1}+\frac{b^3}{a+1}\right)+5\left(\frac{1^2}{a}+\frac{1^2}{b}\right)\overset{Bergstrom\,\&\,Holder}{\geq}$$

$$\geq\frac{4(a+b)^3}{2(a+b+2)}+\frac{5(1+1)^2}{a+b}\overset{a+b=t>0}{=}\frac{2t^3}{t+2}+\frac{20}{t^2}$$

$$We\ \ need\ to\ show\frac{2t^3}{t+2}+\frac{20}{t^2}\geq 14\ or,$$

$$\frac{t^3}{t+2}+\frac{10}{t^2}\geq 7\ \ or\ t^4+10t+20\geq 7t^2+14t\ or,$$

$$t^4-7t^2-4t+20\geq 0\ or$$

$$(t-2)^2(t^2+4t+5)\geq 0,true$$

$$Equality\ holds\ for\ t=a+b=2\ or\ \ a=b=1$$