ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ then:

$$2\left(\frac{a^3}{(b+1)^2}+\frac{b^3}{(a+1)^2}\right)+\frac{1}{a}+\frac{1}{b}\ge 3$$

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Solution by Tapas Das-India

$$2\left(\frac{a^3}{(b+1)^2}+\frac{b^3}{(a+1)^2}\right)+\frac{1}{a}+\frac{1}{b}\overset{Radon\,\&Bergstrom}{\ge}$$

$$\ge\frac{2(a+b)^3}{(a+b+2)^2}+\frac{(1+1)^2}{a+b}\overset{a+b=t>0}{=}\frac{2t^3}{(t+2)^2}+\frac{4}{t}$$

We need to show:

$$\frac{2t^3}{(t+2)^2}+\frac{4}{t}\ge 3\ or\ ,$$

$$\frac{2t^4+4t^2+16t+16}{3t^3+12t^2+12t}\ge 3\ or,$$

$$2t^4-3t^3-8t^2+4t+16\ge 0\ or,$$

$$(t-2)^2(2t^2+5t+4)\ge 0\ true$$

$$Equality\ holds\ for\ t=a+b=2 or\ a=b=1$$