ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ and $a^4 + b^4 + c^4 = 3$ then prove that:

$$\frac{1}{4-a} + \frac{1}{4-b} + \frac{1}{4-c} \le 1$$

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We have :

$$\frac{1}{4-a} \overset{?}{\le} \frac{a^4+11}{36} \Leftrightarrow 8 - 11a + 4a^4 - a^5 \overset{?}{\ge} 0 \Leftrightarrow (1 - 2a + a^2)(8 + 5a + 2a^2 - a^3) \overset{?}{\ge} 0,$$

which is true for all $a \le 2$. Therefore :

$$\frac{1}{4-a} + \frac{1}{4-b} + \frac{1}{4-c} \le \frac{a^4 + b^4 + c^4 + 3.11}{36} = 1.$$

Equality holds iff $a = b = c = 1$.