ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0, ab + bc + ca + 2abc = 1$ then:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq 4(a+b+c)$$

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$$ab+bc+ca+2abc=1. Let\ a=\frac{x}{y+z}, b=\frac{y}{z+x}, c=\frac{z}{x+y}$$

$$We\ need\ to\ show:$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq 4(a+b+c)\ or\ \frac{y+z}{x}+\frac{z+x}{y}+\frac{x+y}{z} \geq 4\left(\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}\right)$$

$$4\left(\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}\right) \overset{AM-HM}{\leq} 4\left(\frac{1}{4}\left(\frac{x}{y}+\frac{x}{z}\right)+\frac{1}{4}\left(\frac{y}{z}+\frac{y}{x}\right)+\frac{1}{4}\left(\frac{z}{x}+\frac{z}{y}\right)\right)=$$

$$=\frac{y+z}{x}+\frac{z+x}{y}+\frac{x+y}{z}$$

$$Equality\ holds\ for\ x=y=z\ or, a=b=c=\frac{1}{2}.$$