ROMANIAN MATHEMATICAL MAGAZINE

If $x,y>0$, $x+y=8$ then:

$$\frac{\sqrt{x}+\sqrt{y}}{8+\sqrt{xy}}\geq\frac{1}{3}$$

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$$x+y=8\ or\ ,2\sqrt{xy}\overset{AM-GM}{\leq}8\ or,\sqrt{xy}\leq 4\ (1)$$

$$We\ need\ to\ show:$$

$$\frac{\sqrt{x}+\sqrt{y}}{8+\sqrt{xy}}\geq\frac{1}{3}\ or\ 3\left(\sqrt{x}+\sqrt{y}\right)\geq 8+\sqrt{xy}\ or,$$

$$9\left(x+y+2\sqrt{xy}\right)\overset{Squaring}{\geq}64+xy+16\sqrt{xy}$$

$$or\ 72+18\sqrt{xy}\overset{x+y=8}{\geq}64+xy+16\sqrt{xy}$$

$$xy-2\sqrt{xy}-8\leq 0\ or\ \left(\sqrt{xy}-4\right)\left(\sqrt{xy}+2\right)\leq 0$$

$$True\ as\ \sqrt{xy}\leq 4\ \left(from\ (1)\right)$$

Equality holds for x=y=4.