ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\sum \frac{\frac{c}{a}\sqrt[2025]{b^{2025}+c^{2025}}+\frac{b}{a}\sqrt[2025]{c^{2025}+a^{2025}}}{c\sqrt[2024]{\frac{b^{2024}+c^{2024}}{a^{2024}+b^{2024}}}+b\sqrt[2024]{\frac{c^{2024}+a^{2024}}{a^{2024}+b^{2024}}}}\ge 3\sqrt[2025]{2}$$

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Walter Janous inequality: a',b',c' and x',y',z' be positive real numbers then:

$$\frac{x'}{y'+z'}(b'+c')+\frac{y'}{z'+x'}(c'+a')+\frac{z'}{x'+y'}(b'+a')\ge\sqrt{3(a'b'+b'c'+c'a')}\quad(1)$$

$$\text{Let } x'=\frac{\sqrt[2024]{(a^{2024}+b^{2024})}}{a},\ y'=\frac{\sqrt[2024]{(c^{2024}+b^{2024})}}{b},\ z'=\frac{\sqrt[2024]{(a^{2024}+c^{2024})}}{c}$$

$$\text{and } a'=\frac{\sqrt[2025]{(b^{2025}+a^{2025})}}{a},\ b'=\frac{\sqrt[2025]{(b^{2025}+c^{2025})}}{b},\ c'=\frac{\sqrt[2025]{(c^{2025}+a^{2025})}}{c}$$

$$\sum a'b'=\sum\left(\frac{\sqrt[2025]{(b^{2025}+a^{2025})}}{a}\cdot\frac{\sqrt[2025]{(b^{2025}+c^{2025})}}{b}\right)\overset{AM-GM}{\ge}$$

$$\ge\sum\frac{\sqrt[2025]{2(ab)^{\frac{2025}{2}}.2(bc)^{\frac{2025}{2}}}}{bc}=2^{\frac{2}{2025}}\sum\frac{\sqrt{abbc}}{bc}=2^{\frac{2}{2025}}\sum\sqrt{\frac{a}{c}}\quad(2)$$

$$\sum \frac{\frac{c}{a}\sqrt[2025]{b^{2025}+c^{2025}}+\frac{b}{a}\sqrt[2025]{c^{2025}+a^{2025}}}{c\sqrt[2024]{\frac{b^{2024}+c^{2024}}{a^{2024}+b^{2024}}}+b\sqrt[2024]{\frac{c^{2024}+a^{2024}}{a^{2024}+b^{2024}}}}=$$

$$=\sum\frac{\sqrt[2024]{(a^{2024}+b^{2024})}}{a}\cdot\left(\frac{\frac{\sqrt[2025]{(b^{2025}+c^{2025})}}{b}+\frac{\sqrt[2025]{(c^{2025}+a^{2025})}}{c}}{\frac{\sqrt[2024]{(c^{2024}+b^{2024})}}{b}+\frac{\sqrt[2024]{(a^{2024}+c^{2024})}}{c}}\right)=$$

$$=\sum\frac{x'(b'+c')}{y'+z'}\overset{(1)}{\ge}\sqrt{3(a'b'+b'c'+c'a')}\overset{(2)}{\ge}\sqrt{3.2^{\frac{2}{2025}}\sum\sqrt{\frac{a}{c}}}\overset{AM-GM}{\ge}$$

$$\ge\sqrt{3.2^{\frac{2}{2025}}.3}=3\sqrt[2025]{2}$$

Equality holds for $a=b=c$.