

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\frac{a^{2025}}{b} + \frac{b^{2025}}{c} + \frac{c^{2025}}{a} \geq \frac{(ab^{2023} + bc^{2023} + ca^{2023})^{2025}}{(a^{2024} + b^{2024} + c^{2024})^{2024}}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\begin{aligned} & \left(\frac{a^{2025}}{b} + \frac{b^{2025}}{c} + \frac{c^{2025}}{a} \right) (b^{2024} + c^{2024} + a^{2024})^{2024} \geq \\ \stackrel{\text{Holder}}{\geq} & \left(\sum^{2025} \sqrt{\frac{a^{2025}}{b} \cdot b^{(2024 \times 2024)}} \right)^{2025} = \left(\sum^{2025} \sqrt{\frac{a^{2025}}{b} \cdot b^{4096576}} \right)^{2025} = \\ & = \left(\sum^{2025} \sqrt{\frac{a^{2025}}{b} \cdot b^{4096575} \cdot b} \right)^{2025} = \left(\sum^{2025} \sqrt{a^{2025} \cdot b^{4096575}} \right)^{2025} = \\ & = \left(\sum^{2025} \sqrt{(ab^{2023})^{2025}} \right)^{2025} = (ab^{2023} + bc^{2023} + ca^{2023})^{2025} \end{aligned}$$

$$\frac{a^{2025}}{b} + \frac{b^{2025}}{c} + \frac{c^{2025}}{a} \geq \frac{(ab^{2023} + bc^{2023} + ca^{2023})^{2025}}{(a^{2024} + b^{2024} + c^{2024})^{2024}}$$

Equality holds for $a = b = c$.