ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a} \geq \frac{(a+b+c)^{2025}}{a\sqrt[2024]{b}+b\sqrt[2024]{c}+c\sqrt[2024]{a}}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=\sum\frac{a}{b}=\sum\frac{a^{2025}}{ba^{2024}}$$

$$\left(\sum\frac{a^{2025}}{ba^{2024}}\right)\left(\sum a\sqrt[2024]{b}\right)^{2024} \overset{Holder}{\geq}$$

$$\geq \left(\sum\sqrt[2025]{\frac{a^{2025}}{ba^{2024}}\cdot a^{2024}b}\right)^{2025} = (a+b+c)^{2025}$$

$$\left(\sum\frac{a^{2025}}{ba^{2024}}\right) \geq \frac{(a+b+c)^{2025}}{\left(\sum a\sqrt[2024]{b}\right)^{2024}}$$

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=\sum\frac{a}{b}=\sum\frac{a^{2025}}{ba^{2024}}=$$

$$=\frac{(a+b+c)^{2025}}{\left(\sum a\sqrt[2024]{b}\right)^{2024}}=\frac{(a+b+c)^{2025}}{a\sqrt[2024]{b}+b\sqrt[2024]{c}+c\sqrt[2024]{a}}$$

Equality holds for $a=b=c$.