*Number 38* *AUTUMN 2025*

# R M M

## ROMANIAN MATHEMATICAL MAGAZINE

Founding Editor
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*Available online*
*www.ssmrmh.ro*

ISSN-L 2501-0099

# PROBLEMS FOR JUNIORS

**JP.556. In acute $\Delta ABC$ the following relationship holds:**

$$\sum_{cyc} \sin^2 A(\cos B + \cos C) \le 2 + \frac{1}{8}\left(13\frac{r}{R} - 15\left(\frac{r}{R}\right)^2 - 6\left(\frac{r}{R}\right)^3\right)$$

*Proposed by Marian Ursărescu - Romania*

**JP.557. In $\Delta ABC$ the following relationship holds:**

$$\left(\frac{b}{c} + \frac{c}{b}\right)\cos^2\frac{A}{2} + \left(\frac{a}{c} + \frac{c}{a}\right)\cos^2\frac{B}{2} + \left(\frac{a}{b} + \frac{b}{a}\right)\cos^2\frac{C}{2} \le \frac{3}{2}\left(\frac{R}{r} + 1\right)$$

*Proposed by Marian Ursărescu - Romania*

**JP.558. If $a, b, c > 0; x \ge 0$ then:**

$$\sum_{cyc} \frac{(a^3 + x)(b^3 + x)}{ac^2 + x} \ge \sum_{cyc} \frac{(ba^2 + x)(cb^2 + x)}{c^3 + x}$$

*Proposed by Daniel Sitaru - Romania*

**JP.559. If $x, y, z > 0$ then:**

$$9\sum_{cyc}\left(\frac{2x + y + z}{x^2 + 2}\right)^2 \le 2\sum_{cyc}(x^2 + 2)(y^2 + 2)$$

*Proposed by D.M. Bătineţu - Giurgiu, Daniel Sitaru - Romania*

**JP.560. If $x, y, z > 0; xyz \le 1$ then:**

$$\sum_{cyc}\left(\frac{1}{x^7} + \frac{1}{y^5} + \frac{1}{z^3}\right) \ge \frac{3(x + y + z)}{xyz}$$

*Proposed by Daniel Sitaru - Romania*

**JP.561. Solve for real numbers:**

$$\log_{2\sqrt{8+2\sqrt{15}}}(x^2 + x + 2) = \log_{\sqrt{4+\sqrt{15}}}(x^2 + x + 1)$$

*Proposed by Marian Ursărescu and Florică Anastase - Romania*

**JP.562.** If $a,b,c\in(0,1)$ and $x,y,z>0$ such that $a=(bc)^x$, $b=(ca)^y$, $c=(ab)^z$ and $xyz=1$ then holds:

$$\sqrt[n]{\sum_{cyc}a^n(a^n+y+2)^{2n-1}}\geq 6\cdot\sqrt[3]{abc},n\in\mathbb{N}^*,n\geq 2$$

*Proposed by Marian Ursărescu and Florică Anastase - Romania*

**JP.563.** In acute triangle $ABC$, $A',B',C'$ are symmetric points $A,B,C$ to the sides $BC,AC$ and $AB$ respectively. Prove that:

$$\frac{\sigma[A'B'C']}{\sigma[ABC]}=4\left(\frac{r}{R}\right)^2+8\cdot\frac{r}{R}-1$$

where $\sigma[ABC]$ is area of $ABC$.

*Proposed by Marian Ursărescu and Florică Anastase - Romania*

**JP.564.** If $x,y,z>0;x^2+y^2+z^2=\frac{3}{4}$ then:

$$4(x+y+z)+2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\geq 18$$

*Proposed by Daniel Sitaru - Romania*

**JP.565.** If $a,b>0$ then:

$$3\sqrt[3]{a}+5\sqrt[5]{a}+3\sqrt[3]{b}+5\sqrt[5]{b}\leq 4\sqrt{a}+4\sqrt{b}+8$$

*Proposed by Daniel Sitaru - Romania*

**JP.566.** In $\Delta ABC$ the following relationship holds:

$$8\leq\sum\left(\frac{b}{c}+\frac{c}{b}\right)\frac{a^2}{m_a^2}\leq\frac{4R}{r}\left(\frac{R}{r}-1\right)$$

*Proposed by Marin Chirciu - Romania*

**JP.567.** Let $ABC$ be a triangle with inradius $r$ and circumradius $R$. Prove that:

$$\frac{9\sqrt{3}}{2}\cdot\frac{r^2}{R^2}\leq\sin^3A+\sin^3B+\sin^3C\leq\frac{9\sqrt{6}}{8}\sqrt{1-\frac{r}{R}}$$

*Proposed by George Apostolopoulos - Greece*

**JP.568.** Let $ABC$ be a triangle, with inradius $r$, and circumradius $R$. Prove that:

$$4\leq\sec^2\frac{A}{2}+\sec^2\frac{B}{2}+\sec^2\frac{C}{2}\leq 2\left(\frac{R}{2r}\right)^4+2$$

*Proposed by George Apostolopoulos - Greece*

**JP.569. In $\Delta ABC$ prove that:**

$$\left|\frac{a^2-b^2}{ab}\right|+\left|\frac{b^2-c^2}{bc}\right|+\left|\frac{c^2-a^2}{ca}\right|<\frac{3R}{r}$$

*Proposed by Ertan Yildirim - Turkiye*

**JP.570. In $\Delta ABC$ the following relationship holds:**

$$\frac{12R^2p}{R}\le\sum w_a\sqrt{\frac{b^2+c^2}{2}}\le\frac{9R^2}{2}\sqrt{3}$$

*Proposed by Marin Chirciu - Romania*

# PROBLEMS FOR SENIORS

**SP.556. We consider the function $f:(0,\infty)\to\mathbb{R}, f(x)=\frac{\sqrt{3x+\sin x}}{x}$. Prove that it is integrable and prove the inequality:**

$$\int_{\varepsilon}^{8} f(x)dx<10+\ln 2$$

**where $1>\varepsilon>0$ and $\varepsilon\to 0$.**

*Proposed by Adalbert Kovacs – Romania*

**SP.557. Prove the following inequality:**

$$\int_{\frac{1}{2024}}^{3}\frac{\sin x}{x}dx<2+\ln 3$$

*Proposed by Adalbert Kovacs – Romania*

**SP.558. In $\Delta ABC, O$ - circumcenter. If the bisector from angle $A$, altitude from angle $B$ and $CO$ circumcevian are in concurrence, then holds:**

$$\sqrt[3]{-1+3\frac{r}{R}-\frac{3}{2}\left(\frac{r}{R}\right)^2}\le\cos A\le\sqrt[3]{\frac{1}{2}\left(\frac{r}{R}\right)^2}$$

*Proposed by Marian Ursărescu - Romania*

**SP.559. In acute $\Delta ABC, AD, BE, CF$ - symmedians. Prove that:**

$$\frac{r^3}{R}\le\frac{AF\cdot BD\cdot CE}{AB+BC+CA}\le\frac{Rr}{4}$$

*Proposed by Marian Ursărescu - Romania*

**SP.560. For $k \in \mathbb{N}$ fixed and $\alpha > 0$ evaluate:**

$$L = \lim_{n\to\infty} \frac{1}{\sqrt{n^\alpha}} \cdot \left(\prod_{i=1}^{k} \frac{n+k+i}{n+i}\right)^{n^\alpha}$$

*Proposed by Marian Ursărescu and Florică Anastase - Romania*

**SP.561. Let be the function $f : [0,1] \to \mathbb{R}$ integrable such that $f(1) = 1$ and**

$$\int_x^y f(t)dt = \frac{1}{2}(yf(y) - xf(x)), \quad \forall x, y \in [0,1]$$

**Find:**

$$I = \int_0^{\frac{\pi}{4}} f(x) \cdot \tan^2 x dx.$$

*Proposed by Marian Ursărescu and Florică Anastase - Romania*

**SP.562. If $0 < a \leq b < \frac{\pi}{2}$ then:**

$$\int_a^b \frac{2 + \sqrt{2}(\sin x + \cos x)}{\sin\left(\frac{\pi}{4} + x\right)} \geq 4(b - a)$$

*Proposed by Daniel Sitaru - Romania*

**SP.563. If $a, b, c > 0, a + b + c = 3$, then:**

$$\sum \frac{2a^4}{3a^2 - 2a + 5} \geq 1$$

*Proposed by Marin Chirciu - Romania*

**SP.564. If $a, b, c > 0, (a + b - 1)^2 = ab$ and $\lambda \geq 0$ then:**

$$\frac{1}{ab} + \frac{1}{a^2 + b^2} + \frac{\lambda\sqrt{ab}}{a + b} \geq 1 + \sqrt{\lambda}$$

*Proposed by Marin Chirciu - Romania*

**SP.565. If $a_1, a_2, \ldots, a_n > 0, a_1 + a_2 + \ldots + a_n = n$ then:**

$$\left(1 + \frac{1}{a_1}\right)^{a_2^2}\left(1 + \frac{1}{a_2}\right)^{a_3^2} \cdot \ldots \cdot \left(1 + \frac{1}{a_n}\right)^{a_1^2} \geq 2^n$$

*Proposed by Marin Chirciu - Romania*

**SP.566. If $\lambda > 0$ then find:**

$$\int_0^1 \frac{(x^2e^x + (\lambda + 1)x + 1)e^x}{\lambda + xe^x} dx$$

*Proposed by Marin Chirciu - Romania*

**SP.567. Prove that $\frac{3}{2}$ is the largest positive value of the constant $k$ such that the inequality**

$$(a+k)^2+(b+k)^2+(c+k)^2+(d+k)^2+(e+k)^2+(f+k)^2 \geq 6(1+k)^2$$

**holds whenever $a \geq b \geq 1 \geq c \geq d \geq e \geq f \geq 0$ satisfying**

$$ab+bc+cd+de+ef+fa=6$$

*Proposed by Vasile Cîrtoaje - Romania*

**SP.568. Let $a_1 \geq 1 \geq a_2 \geq \ldots \geq a_n \geq 0$ such that $a_1+a_2+\ldots a_n=n$. Prove that:**

$$a_1a_2+a_2a_3+\ldots+a_na_1 \leq n$$

*Proposed by Vasile Cîrtoaje - Romania*

**SP.569. Let $a,b,c$ be positive real numbers such that at most one of them is greater than 1 and $abc=1$. Prove that:**

$$\frac{11(b^2+c^2)-10a^2}{b+c}+\frac{11(c^2+a^2)-10b^2}{c+a}+\frac{11(a^2+b^2)-10c^2}{a+b} \leq 18$$

*Proposed by Vasile Cîrtoaje - Romania*

**SP.570. Let $a,b>1$ fixed. Solve the equation:**

$$\left(\frac{x}{ab^2}\right)^{\log_{\sqrt{a}} x}=\left(\frac{x}{a^2b}\right)^{\log_{\sqrt{b}} x}$$

*Proposed by Marin Chirciu - Romania*

# UNDERGRADUATE PROBLEMS

**UP.556. Let be $f:[a,b]\to\mathbb{R}; 0<a\leq b, f$ continuous, convex and**

$$\int_0^a f(x)dx=a; \quad \int_0^b f(x)dx=2b$$

**Prove that:**

$$\int_0^{\frac{a+b}{2}} f(x)dx \leq a+b$$

*Proposed by Daniel Sitaru - Romania*

**UP.557. Prove:**

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta \arccos\left(\frac{\cos^6 2\theta}{1+\sin^2\theta}\right)}{1+\sin^2\theta}d\theta=\frac{3}{9}\zeta(2)$$

*Proposed by Said Attaoui – Algeria*

**UP.558.Prove that 3 is the greatest positive value of $k$ such that:**

$$\frac{1}{a+k}+\frac{1}{b+k}+\frac{1}{c+k}+\frac{1}{d+k}+\frac{1}{e+k}\geq\frac{5}{1+k}$$

**for any $a\geq b\geq c\geq d\geq 1\geq e\geq 0$ satisfying $ab+bc+de+ea=5$.**

*Proposed by Vasile Cîrtoaje - Romania*

**UP.559. For given $n\geq 2$, prove that $n-1$ is the smaller positive value of the constant $k$ such that:**

$$a_1+a_2+\ldots+a_n\geq\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}$$

**for any positive numbers $a_i$ with $a_1\leq a_2\leq\ldots\leq a_n$ and $a_1^k a_n\geq 1$.**

*Proposed by Vasile Cî rtoaje - Romania*

**UP.560.Prove:**

$$\int_0^\infty\int_0^\infty\frac{(x+y)e^{-(x+y)}}{1-e^{-(x+y)}}dxdy=2\zeta(3)$$

*Proposed by Said Attaoui – Algeria*

**UP.561. Prove:**

$$\frac{2}{3}\int_0^\pi\frac{x\sin(x)}{1+\cos^2(x)}dx=\zeta(2)$$

*Proposed by Said Attaoui – Algeria*

**UP.562. Prove:**

$$\int_0^\infty\frac{\sin(2x)}{x}\log(x)dx=-\frac{\pi}{2}(\log 2+\gamma)$$

**Is there a way to prove that?**

$$\int_0^1\frac{\log(\frac{1-x}{x})}{x(1-x)}\cos\Big(\frac{1-2x+2x^2}{2x(1-x)}\Big)\sin\Big(\frac{1-2x}{2x(1-x)}\Big)dx=-\frac{\pi\gamma}{2}$$

**where $\gamma$ design the Euler - Mascheroni constant given by $\Gamma'(1)=\gamma=0.557215...$ and $\Gamma(\alpha)=\int_0^\infty t^{\alpha-1}e^{-t}dt$ is the gamma function.**

*Proposed by Said Attaoui – Algeria*

**UP.563. Calculate the integral:**

$$\int_1^\infty\frac{\sqrt{x}\ln^2 x}{(x+1)(x^2+1)}dx$$

*Proposed by Vasile Mircea Popa - Romania*

**UP.564. Calculate the integral:**

$$\int_0^8 \frac{\arctan(x)}{\sqrt{x^4 - x^2 + 1}} dx$$

*Proposed by Vasile Mircea Popa - Romania*

**UP.565. Evaluate:**

$$\sum_{q=0}^{\infty} (-1)^q \left( \frac{q}{(2q+1)(q+1)} \right)^2$$

*Proposed by Said Attaoui – Algeria*

**UP.566. If $0 < a \leq b < 1$ then:**

$$\int_a^b \int_a^b \int_a^b \sqrt{\frac{xyz}{(1-x)(1-y)(1-z)}} dxdydz \geq (b^2 - a^2)^3$$

*Proposed by Daniel Sitaru - Romania*

**UP.567. If $-2 < a \leq b < 2$ then:**

$$\int_a^b \int_a^b \frac{x}{x^2 - xy + 1} dxdy \leq \left( \frac{2-a}{2-b} \right)^{ab}$$

*Proposed by Daniel Sitaru - Romania*

**UP.568. Let $a \neq 0, a + b \neq 0$. Find all the functions $f : \mathbb{R} \to \mathbb{R}$ such that:**

$$f(2) = 1 \text{ and } af(f(x)) + bf(x) = a + b, \forall x \in \mathbb{R}.$$

*Proposed by Marin Chirciu - Romania*

**UP.569. Let $\lambda \in \mathbb{N}^*$ fixed. Evaluate:**

$$\lim_{n \to \infty} \{ \sqrt{\lambda^2 n^2 + (2\lambda - 1)n + 1} \}$$

*Proposed by Marin Chirciu - Romania*

**UP.570. Solve for real numbers the equation:**

$$\sqrt{18 + x} + \sqrt{7 - x} = x^2 - 11x + 25$$

*Proposed by Marin Chiricu - Romania*

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