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In ΔABC the following relationship holds:

$$\frac{1}{w_a sin\frac{A}{2}}+\frac{1}{w_b sin\frac{B}{2}}+\frac{1}{w_c sin\frac{C}{2}}=4\left(\frac{sin\frac{A}{2}}{w_a}+\frac{sin\frac{B}{2}}{w_b}+\frac{sin\frac{C}{2}}{w_c}\right)$$

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$$\frac{1}{w_a\sin\left(\frac{A}{2}\right)}=\frac{1}{\frac{2bc}{b+c}.\cos\left(\frac{A}{2}\right).\sin\left(\frac{A}{2}\right)}=\frac{b+c}{bcsin(A)}=\frac{2R(sinB+sinC)}{4R^2.\sin(A).\sin(B).\sin(C)}=$$

$$=\frac{\sin(B)+\sin(C)}{2R.\frac{F}{2R^2}}=\frac{R.\sin(B)+R.\sin(C)}{F}=\frac{b+c}{2F}$$

$$\sum_{cyc}\frac{1}{w_a sin\frac{A}{2}}=\sum_{cyc}\frac{b+c}{2F}=\frac{4s}{2F}=\frac{2s}{sr}=\frac{2}{r}\ (LHS)$$

$$w_a=\frac{2bc}{b+c}.cos\frac{A}{2}\ ;\ h_a=\frac{2F}{a}=\frac{2}{a}.\frac{1}{2}bcsinA=\frac{bc}{a}.sinA$$

$$\frac{w_a}{h_a}=\frac{\frac{2bc}{b+c}.cos\frac{A}{2}}{\frac{bc}{a}.sinA}=\frac{a}{(b+c).sin\frac{A}{2}}\rightarrow w_a=\frac{a.h_a}{(b+c).sin\frac{A}{2}}$$

$$4.\frac{sin\frac{A}{2}}{w_a}=4.\frac{(b+c).sin^2\frac{A}{2}}{a.h_a}=4.\frac{b+c}{a}.\frac{sin^2\frac{A}{2}}{h_a}=4.\frac{cos\frac{B-C}{2}}{sin\frac{A}{2}}.\frac{sin^2\frac{A}{2}}{h_a}=4.\frac{cos\frac{B-C}{2}.sin\frac{A}{2}}{h_a}$$

$$Here\ \boxed{\frac{\cos\left(\frac{B-C}{2}\right)}{\sin\left(\frac{A}{2}\right)}=\frac{b+c}{a}}\ (Mollweides\ \ formula)$$

$$\cos\left(\frac{B-C}{2}\right).\sin\left(\frac{A}{2}\right)=\frac{1}{2}\left(\sin\left(\frac{A}{2}+\frac{B-C}{2}\right)+\sin\left(\frac{A}{2}-\frac{B-C}{2}\right)\right)=$$

$$\frac{1}{2}\left(\sin\left(\frac{\pi}{2}-C\right)+\sin\left(\frac{\pi}{2}-B\right)\right)=\frac{1}{2}(\cos(B)+\cos(C))$$

$$4.\frac{\sin\left(\frac{A}{2}\right)}{w_a}=4.\frac{\cos\left(\frac{B-C}{2}\right).\sin\left(\frac{A}{2}\right)}{h_a}=\frac{2}{h_a}(\cos(B)+\cos(C))$$

$$4\sum\frac{sin\frac{A}{2}}{w_a}=\frac{2}{h_a}(\cos(B)+\cos(C))+\frac{2}{h_b}(\cos(A)+\cos(C))+\frac{2}{h_C}(\cos(B)+\cos(A))=$$

$$=\frac{a}{F}(\cos(B)+\cos(C))+\frac{b}{F}(\cos(A)+\cos(C))+\frac{C}{F}(\cos(B)+\cos(A))=$$

$$=\frac{1}{F}\big((a.\cos(A)+b.\cos(A))+(c.\cos(A)+a.\cos(C))+(b.\cos(C)+c.\cos(B))\big)=$$

$$=\frac{1}{F}(a+b+c)=\frac{2S}{F}=\frac{2S}{Sr}=\frac{2}{r}(RHS)\quad Proved$$

$$Here\ \boxed{\begin{array}{l}a=b.\cos(C)+c.\cos(B)\\ b=c.\cos(A)+a.\cos(C)\\ c=a.\cos(B)+b.\cos(A)\end{array}}$$