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In ΔABC the following relationship holds:

$$\sqrt{\frac{g_a^2-h_a^2}{w_a^2-h_a^2}}+\sqrt{\frac{g_b^2-h_b^2}{w_b^2-h_b^2}}+\sqrt{\frac{g_c^2-h_c^2}{w_c^2-h_c^2}}=2$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$g_a^2=s(s-a)-\frac{(s-a)(b-c)^2}{a},h_a^2=s(s-a)-\frac{s(s-a)(b-c)^2}{a^2},w_a^2=s(s-a)-\frac{s(s-a)}{(b+c)^2}(b-c)^2$$

(Reference: Bogdan Fustei & Mohamed Amine Ben Ajiba- ABOUT NAGEL AND GERGONNE'S CEVIANS-www.ssmrmh.ro)

$$g_a^2-h_a^2=s(s-a)-\frac{(s-a)(b-c)^2}{a}-s(s-a)+\frac{s(s-a)(b-c)^2}{a^2}=$$
$$=\frac{(s-a)(b-c)^2}{a^2}(s-a)=\frac{(s-a)^2(b-c)^2}{a^2}$$

$$w_a^2-h_a^2=s(s-a)-\frac{s(s-a)}{(b+c)^2}(b-c)^2-s(s-a)+\frac{s(s-a)(b-c)^2}{a^2}=$$

$$=\frac{s(s-a)(b-c)^2}{a^2}\left(1-\frac{a^2}{(b+c)^2}\right)=$$

$$=\frac{s(s-a)(b-c)^2}{a^2}\frac{(a+b+c)(b+c-a)}{(b+c)^2}=\frac{2s^2(s-a)(b-c)^2}{a^2}\frac{2(s-a)}{(b+c)^2}$$

Using above result we get:

$$\frac{g_a^2-h_a^2}{w_a^2-h_a^2}=\frac{(b+c)^2}{4s^2},\sqrt{\frac{g_a^2-h_a^2}{w_a^2-h_a^2}}=\frac{b+c}{2s}$$

$$\sqrt{\frac{g_a^2-h_a^2}{w_a^2-h_a^2}}+\sqrt{\frac{g_b^2-h_b^2}{w_b^2-h_b^2}}+\sqrt{\frac{g_c^2-h_c^2}{w_c^2-h_c^2}}=\sum\sqrt{\frac{g_a^2-h_a^2}{w_a^2-h_a^2}}=$$

$$=\sum\frac{b+c}{2s}=\frac{2(a+b+c)}{2s}=\frac{4s}{2s}=2$$