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In $\triangle ABC$ the following relationship holds:

$$\tan\frac{A}{2}\tan\frac{B}{2}=1-\frac{2r}{h_c}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$tan\frac{A}{2}tan\frac{B}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} =$$

$$= \frac{s-c}{s} = 1 - \frac{c}{s} = 1 - \frac{\frac{F}{s}}{\frac{2F}{c}} \cdot 2 = 1 - \frac{2r}{h_c}$$