

# ROMANIAN MATHEMATICAL MAGAZINE

If in  $\triangle ABC$ ,  $A = 108^\circ$ ,  $B = C = 36^\circ$  then:

$$w_b = 2m_a$$

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Using the known results:

$$\sin 108^\circ = \cos 18^\circ = \frac{1}{4}\sqrt{10 + 2\sqrt{5}}, \sin 36^\circ = \frac{1}{4}\sqrt{10 - 2\sqrt{5}}$$

$$a = 2R \sin A = 2R \sin 108^\circ = \frac{R}{2}\sqrt{10 + 2\sqrt{5}}, b = c = 2R \sin 36^\circ = \frac{R}{2}\sqrt{10 - 2\sqrt{5}}$$

$$\begin{aligned} 2b^2 + 2c^2 - a^2 &= 4b^2 - a^2 = 4 \cdot \frac{R^2}{4} \left( \sqrt{10 - 2\sqrt{5}} \right)^2 - \frac{R^2}{4} \left( \sqrt{10 + 2\sqrt{5}} \right)^2 = \\ &= \frac{R^2}{4} (30 - 10\sqrt{5}) = \frac{R^2}{4} (5 - \sqrt{5})^2, 2m_a = \sqrt{2b^2 + 2c^2 - a^2} = \frac{R}{2} (5 - \sqrt{5}) \quad (A) \end{aligned}$$

$$\begin{aligned} \cos \frac{B}{2} = \cos 18^\circ &= \frac{1}{4}\sqrt{10 + 2\sqrt{5}}, 2ac = 2 \cdot \frac{R}{2}\sqrt{10 + 2\sqrt{5}} \cdot \frac{R}{2}\sqrt{10 - 2\sqrt{5}} = \\ &= \frac{R^2}{2}\sqrt{100 - 20} = \frac{R^2}{2}4\sqrt{5} = 2\sqrt{5}R^2 \end{aligned}$$

$$\begin{aligned} a + c &= 2R(\sin A + \sin C) = 4R \sin \frac{A+C}{2} \cos \frac{A-C}{2} = \\ &= 4R \sin 72^\circ \cos 36^\circ = 4R \cdot \frac{1}{4}\sqrt{10 + 2\sqrt{5}} \frac{\sqrt{5} + 1}{4} = \frac{R}{4}(\sqrt{5} + 1)\sqrt{10 + 2\sqrt{5}} \end{aligned}$$

Using above result we get:

$$w_b = \frac{2ac}{a+c} \cos \frac{B}{2} = \frac{2\sqrt{5}R}{\sqrt{5} + 1} = \frac{R}{2}(5 - \sqrt{5}) \quad (B)$$

from (A) & (B) we get  $w_b = 2m_a$