

ROMANIAN MATHEMATICAL MAGAZINE

If in $\triangle ABC$, $A = 3B$ then:

$$(a - b)^2(a + b) = bc^2$$

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$$A + B + C = \pi \text{ or, } C = \pi - (A + B) = \pi - (3B + B) = \pi - 4B \\ \sin C = \sin(\pi - 4B) = \sin 4B$$

$$bc^2 = 2R \sin B \cdot 4R^2 \sin^2 C = 8R^3 \sin B \cdot \sin^2 4B \quad (i)$$

$$(a - b)^2(a + b) = (a^2 - b^2)(a - b) = 4R^2(\sin^2 A - \sin^2 B)2R(\sin A - \sin B) \\ \stackrel{A=3B}{=} 8R^3 \sin(A + B) \sin(A - B) (\sin 3B - \sin B) = 8R^3 \sin 4B \sin 2B \cdot 2 \cos 2B \sin B$$

$$= 8R^3(2 \sin 2B \cos 2B) \sin B \sin 4B = 8R^3 \sin B \cdot \sin^2 4B \quad (ii)$$

From (i) & (ii) we get $(a - b)^2(a + b) = bc^2$