ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$R^3+s^3+r^3\ge\frac{27+\sqrt{3}}{2}Rsr$$

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$$We\ will\ show: R^3+r^3\ge\frac{\sqrt{3}}{2}Rsr\ \ (1)$$

$$R^3+r^3\overset{Mitrinovic}{\ge}R\sqrt{3}\cdot\frac{3\sqrt{3}Rr}{4}$$

$$R^3+r^3\ge\frac{9R^2r}{4}$$

$$4R^3-9R^2r+4r^3\ge 0$$

$$4x^3-9x^2+4\overset{\frac{R}{r}=x\ge 2\ Euler}{\ge}0$$

$$(x-2)(4x^2-x-2)\ge 0$$

$$(x-2)\left(x(2x-1)+2(x^2-1)\right)\ge 0\ true\ as\ \ x\ge 2$$

$$s^3=s^2.s\ge\frac{27Rr}{2}.s\ \left(as\ s^2\ge\frac{27Rr}{2}\right)=\frac{27}{2}Rrs\ (2)$$

$$R^3+s^3+r^3\overset{(1)\&(2)}{\ge}\frac{\sqrt{3}}{2}Rsr+\frac{27}{2}Rrs=\frac{27+\sqrt{3}}{2}Rsr$$

Equality holds for an equilateral triangle.