ROMANIAN MATHEMATICAL MAGAZINE

In any non − obtuse Δ ABC, the following relationship holds :

$$\frac{2c}{w_b^2} \geq \frac{1}{2m_a} + \frac{1}{c}$$

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$$\frac{2c}{w_b^2} - \left(\frac{1}{2m_a} + \frac{1}{c}\right) \geq \frac{2c(c+a)^2}{4ca.s(s-b)} - \left(\frac{1}{a} + \frac{1}{c}\right)$$

$$(\because \text{in non} - \text{obtuse } \Delta\, ABC, 4m_a^2 - a^2 = 2(b^2 + c^2 - a^2) \geq 0 \Rightarrow 2m_a \geq a)$$

$$= \frac{2(c+a)^2}{a((c+a)^2 - b^2)} - \frac{c+a}{ca} \overset{?}{\geq} 0 \Leftrightarrow 2c(c+a) \overset{?}{\geq} (c+a)^2 - b^2$$

$$\Leftrightarrow 2c^2 + 2ca \overset{?}{\geq} c^2 + a^2 + 2ca - b^2 \Leftrightarrow b^2 + c^2 \overset{?}{\geq} a^2 \rightarrow \text{true} \because$$

$$\Delta\, ABC \text{ is non} - \text{obtuse} \therefore \frac{2c}{w_b^2} \geq \frac{1}{2m_a} + \frac{1}{c}\ \forall \text{ non} - \text{obtuse } \Delta\, ABC,$$

$$'' ='' \text{ iff } \hat{A} = 90^\circ \text{ (QED)}$$