ROMANIAN MATHEMATICAL MAGAZINE

**In any $\Delta ABC$ the following relationship holds :**

$$2R-r-2\sqrt{R(R-2r)} \le m_a \le 2R-r+2\sqrt{R(R-2r)}$$

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*Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco*

**We have :**

$$4 \le (m_b+m_c)\left(\frac{1}{m_b}+\frac{1}{m_c}\right) = [(m_a+m_b+m_c)-m_a]\left[\left(\frac{1}{m_a}+\frac{1}{m_b}+\frac{1}{m_c}\right)-\frac{1}{m_a}\right]$$

$$\overset{\substack{\text{Leuenberger} \\ m_a \ge h_a \text{ (and analogs)}}}{\le} (4R+r-m_a)\left(\left(\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}\right)-\frac{1}{m_a}\right) = (4R+r-m_a)\left(\frac{1}{r}-\frac{1}{m_a}\right)$$

$$\Leftrightarrow 4rm_a \le (4R+r-m_a)(m_a-r) \Leftrightarrow {m_a}^2-2(2R-r)m_a+r(4R+r) \le 0$$

$$\Leftrightarrow \left(m_a-2R+r+2\sqrt{R(R-2r)}\right)\left(m_a-2R+r-2\sqrt{R(R-2r)}\right) \le 0$$

$$\Leftrightarrow 2R-r-2\sqrt{R(R-2r)} \le m_a \le 2R-r+2\sqrt{R(R-2r)}.$$

**Equality holds iff $\Delta ABC$ is equilateral.**