ROMANIAN MATHEMATICAL MAGAZINE

In the non − obtuse ΔABC, prove that

$$r_a + r \leq \sqrt{7b^2 + 7c^2 - 2bc - 4a^2}$$

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We have:

$$r_a + r = r\left(\frac{s}{s-a} + 1\right) = (b+c).\frac{r}{s-a} = (b+c)\tan\frac{A}{2} \overset{A \leq \frac{\pi}{2}}{\leqslant} b + c \overset{?}{\leqslant} \sqrt{7b^2 + 7c^2 - 2bc - 4a^2}$$

$$0 \overset{?}{\leqslant} 4\left(b^2 + c^2 - a^2\right) + 2(b-c)^2,$$

which is true for non − obtuse ΔABC.

Equality holds iff ABC is isosceles right triangle at A.