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In ΔABC the following relationship holds:

$$\frac{w_a}{h_a}+\frac{w_b}{h_b}+\frac{w_c}{h_c}\le\frac{3}{2}+\frac{3R}{4r}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$\frac{w_a}{h_a}=\frac{2\sqrt{bc.s(s-a)}}{b+c}\frac{2R}{bc}=\frac{4R\sqrt{s(s-a)}}{\sqrt{bc}(2s-a)}=\frac{4R\sqrt{s(s-a)}}{\sqrt{bc}(2(s-a)+a)}\overset{AM-GM}{\le}$$

$$\le\frac{4R\sqrt{s(s-a)}}{\sqrt{bc}2\sqrt{2(s-a).a}}=\frac{2R\sqrt{s}}{\sqrt{2abc}}=\frac{2R\sqrt{s}}{\sqrt{8Rrs}}=\sqrt{\frac{R}{2r}}\ (1)$$

$$\frac{w_a}{h_a}+\frac{w_b}{h_b}+\frac{w_c}{h_c}\overset{(1)}{\le}3\sqrt{\frac{R}{2r}}=3\sqrt{1.\frac{R}{2r}}\overset{AM-GM}{\le}3.\frac{1+\frac{R}{2r}}{2}=\frac{3}{2}+\frac{3R}{4r}$$

$$Equality\ holds\ for\ a=b=c.$$