ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1}{\sqrt{h_a}}+\frac{2}{\sqrt{r_a}}\le\frac{\sqrt{h_a}}{r}$$

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$$We\ need\ to\ show:$$

$$\frac{1}{\sqrt{h_a}}+\frac{2}{\sqrt{r_a}}\le\frac{\sqrt{h_a}}{r}\ or\ \sqrt{\frac{a}{2F}}+2\sqrt{\frac{s-a}{F}}\le\frac{1}{r}\sqrt{\frac{2F}{a}}$$

$$or\ \frac{1}{\sqrt{2F}}\left(\sqrt{a}+2\sqrt{2(s-a)}\right)\le\frac{1}{r}\sqrt{\frac{2F}{a}}\ \ or\left(a+2\sqrt{2(s-a)a}\right)\le\frac{2F}{r}$$

$$or,\left(a+2\sqrt{2(s-a)a}\right)\le\frac{2rs}{r}\ or\left(a+2\sqrt{2(s-a)a}\right)\le 2s\ True$$

$$since:\left(a+2\sqrt{2(s-a)a}\right)\overset{AM-GM}{\le}\ a+\frac{2(s-a)+a}{2}=a+(b+c)=2s$$

Equality holds for an equilateral triangle.