In ΔABC the following relationship holds:

$$\frac{1}{w_a} + \frac{4}{h_a + r_a} \leq \frac{1}{r}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$\frac{1}{w_a} + \frac{4}{h_a + r_a} \overset{w_a \geq h_a}{\leq} \frac{1}{h_a} + \frac{4}{h_a + r_a} \overset{AM-HM}{\leq} \frac{1}{h_a} + \frac{1}{h_a} + \frac{1}{r_a} =$$

$$= \frac{2}{h_a} + \frac{1}{r_a} = \frac{a}{F} + \frac{s-a}{F} = \frac{s}{F} = \frac{1}{r}$$

Equality holds for an equilateral triangle.