ROMANIAN MATHEMATICAL MAGAZINE

In any ΔABC the following relationship holds :

$$\frac{m_a}{w_a}+\frac{w_a}{h_a}+\frac{h_a}{m_a}\le \frac{3}{2}+\frac{3R}{4r}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

Since $h_a \le w_a \le m_a$, then we have

$$\frac{m_a}{w_a}+\frac{w_a}{h_a}+\frac{h_a}{m_a}=\left(\frac{m_a}{w_a}-1\right)\left(1-\frac{w_a}{h_a}\right)+1+\frac{m_a}{h_a}+\frac{h_a}{m_a}\le 1+\frac{m_a}{h_a}+\frac{h_a}{m_a}\le$$

$$\overset{Panaitopol}{\leq} 1+\frac{R}{2r}+1 \overset{Euler}{\leq} \frac{3}{2}+\frac{3R}{4r}$$

as desired. Equality holds iff ΔABC is equilateral.