

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sum \sqrt{\cot A} \cdot \sin A \leq \frac{3R}{4r} \sqrt[4]{3}$$

*Proposed by Kostantinos Geronikolas-Greece*

*Solution by Tapas Das-India*

$$\begin{aligned} \sum \sqrt{\cot A} \cdot \sin A &= \sum \sqrt{\cos A} \cdot \sqrt{\sin A} \stackrel{CBS}{\leq} \sqrt{(\sum \cos A)(\sum \sin A)} = \\ &= \sqrt{\left(1 + \frac{r}{R}\right) \frac{S}{R}} \stackrel{\text{Euler \& Mitrinovic}}{\leq} \sqrt{\left(1 + \frac{1}{2}\right) \frac{3\sqrt{3}R}{2}} = \frac{3}{2} \sqrt[4]{3} = \\ &= \frac{3}{2} \sqrt[4]{3} \cdot \frac{R}{R} \stackrel{\text{Euler}}{\leq} \frac{3}{2} \sqrt[4]{3} \cdot \frac{R}{2r} = \frac{3}{4} \cdot \frac{R}{r} \sqrt[4]{3} \end{aligned}$$

*Equality holds for an equilateral triangle*