# ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC$ the following relationship holds:

$$\sum \frac{\sin^2 A}{h_b^2} \leq \frac{R(R-r)}{8r^4}$$

*Proposed by Kostantinos Geronikolas-Greece*

*Solution by Tapas Das-India*

$$We\ have,: \sum a^4 = 2[(s^2-4Rr-3r^2)^2 - 8r^3(2R+r)] \overset{Gerretsen}{\leq}$$

$$\leq 2[(4R^2)^2 - 8r^3(2R+r)] = 16(2R^4 - 2Rr^3 - r^4)$$

$$We\ will\ show: 16(2R^4 - 2Rr^3 - r^4) \leq 54R^3(R-r)\ (1)$$

$$11R^4 - 27R^3r + 16Rr^3 + 8r^4 \geq 0$$

$$(R-2r)\left((R-2r)(11R^2+17Rr+24r^2)+44r^3\right) \geq 0\ true\ Euler$$

$$\sum \frac{\sin^2 A}{h_b^2} = \sum \frac{\frac{a^2}{4R^2}}{\frac{b^2c^2}{4R^2}} = \frac{1}{a^2b^2c^2}\sum a^4 \overset{(1)}{\leq} \frac{54R^3(R-r)}{16R^2r^2s^2} \overset{Mitrinovic}{\leq}$$

$$\leq \frac{54R^3(R-r)}{16R^2r^2 27r^2} = \frac{R(R-r)}{8r^4}$$

*Equality holds if $\Delta ABC$ is an equilateral one.*