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In ΔABC the following relationship holds:

$$\left(3+\sum\frac{a^2+b^2}{c^2}\right)\left(\sum\sec^2\frac{A}{2}\right)\le\frac{9}{4}\left(\frac{R}{r}\right)^4$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Tapas Das-India

$$\left(\sum\sec^2\frac{A}{2}\right)=3+\sum\tan^2\frac{A}{2}=3+\left(\frac{4R+r}{s}\right)^2-2=$$

$$=1+\left(\frac{4R+r}{s}\right)^2\overset{Euler\ \&\ Mitrinovic}{\le}\left(\frac{R}{R}\right)^2+\frac{\left(\frac{9R}{2}\right)^2}{27r^2}\overset{Euler}{\le}\frac{1}{4}\left(\frac{R}{r}\right)^2+\frac{3}{4}\left(\frac{R}{r}\right)^2=\left(\frac{R}{r}\right)^2$$

$$\left(3+\sum\frac{a^2+b^2}{c^2}\right)=\left(\sum 1+\frac{a^2+b^2}{c^2}\right)=$$

$$=(a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\overset{Leibniz\ \&\ Steining}{\le}9R^2\cdot\frac{1}{4r^2}=\frac{9}{4}\left(\frac{R}{r}\right)^2$$

$$\left(3+\sum\frac{a^2+b^2}{c^2}\right)\left(\sum\sec^2\frac{A}{2}\right)\le\frac{9}{4}\left(\frac{R}{r}\right)^2.\left(\frac{R}{r}\right)^2=\frac{9}{4}\left(\frac{R}{r}\right)^4$$

$$Equality\ holds\ for\ A=B=C.$$