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In ΔABC the following relationship holds:

$$w_a^3 r_a + w_b^3 r_b + w_c^3 r_c \le 3^5 \left(\frac{R}{2}\right)^4$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Tapas Das-India

$$w_a^3 r_a \overset{w_a \le \sqrt{s(s-a)}}{\le} s(s-a)\sqrt{s(s-a)}\frac{F}{s-a} = Fs\sqrt{s}\,\sqrt{s-a}$$

$$w_a^3 r_a + w_b^3 r_b + w_c^3 r_c = \sum w_a^3 r_a \le Fs\sqrt{s}\,\sum\sqrt{s-a} \overset{CBS}{\le}$$

$$\le rs^2\sqrt{s}\sqrt{3(s-a+s-b+s-c)} = rs^3\sqrt{3} \overset{Euler\ \&\ Mitrinovic}{\le}$$

$$\le \frac{R}{2}\cdot\frac{27}{4}R^2\frac{3\sqrt{3}}{2}R\,\sqrt{3} = 3^5\left(\frac{R}{2}\right)^4$$

Equality holds for an equilateral triangle.