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In ΔABC the following relationship holds:

$$\frac{r_a}{m_a}+\frac{r_b}{m_b}+\frac{r_c}{m_c}\geq 4-\frac{2r}{R}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Tapas Das-India

$$\sum\frac{a}{s-a}=\frac{\sum a(s-b)(s-c)}{(s-a)(s-b)(s-c)}=\frac{\sum a(s^2-s(b+c)+bc)}{sr^2}=$$

$$=\frac{2s^3-2s(ab+bc+ca)+3abc}{sr^2}=$$

$$=\frac{2s^3-2s(s^2+r^2+4Rr)+12Rrs}{sr^2}=\frac{2(2R-r)}{r}$$

$$\frac{r_a}{m_a}+\frac{r_b}{m_b}+\frac{r_c}{m_c}\overset{Panaitopol}{\geq}\frac{r_a}{\frac{Rh_a}{2r}}+\frac{r_b}{\frac{Rh_b}{2r}}+\frac{r_c}{\frac{Rh_c}{2r}}=\frac{2r}{R}\left(\frac{r_a}{h_a}+\frac{r_b}{h_b}+\frac{r_c}{h_c}\right)=$$

$$=\frac{r}{R}\sum\frac{a}{s-a}=\frac{r}{R}\frac{2(2R-r)}{r}=4-\frac{2r}{R}$$

Equality holds for an equilateral triangle.