

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sum \frac{1}{2r + r_a} \leq \frac{3}{5r}$$

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$$\text{Let } x = \frac{r}{r_a}, y = \frac{r}{r_b}, z = \frac{r}{r_c} \text{ then } x + y + z = r \sum \frac{1}{r_a} = \frac{r}{r} = 1$$

*We need to show:*

$$\sum \frac{1}{2r + r_a} \leq \frac{3}{5r} \text{ or, } \sum \frac{r}{2r + r_a} \leq \frac{3}{5} \text{ or } \sum \frac{\frac{r}{r_a}}{2\frac{r}{r_a} + 1} \leq \frac{3}{5} \text{ or, } \sum \frac{x}{2x + 1} \leq \frac{3}{5}$$

*Lemma :*

$$\frac{x}{2x + 1} \leq \frac{2 + 9x}{25} \quad \forall x > 0$$

*Proof:*

$$\frac{x}{2x + 1} \leq \frac{2 + 9x}{25} \text{ or } 25x \leq (2x + 1)(2 + 9x) \text{ or } 18x^2 - 12x + 2 \geq 0 \text{ or } \\ 2(3x - 1)^2 \geq 0 \text{ (true)}$$

$$\sum \frac{x}{2x + 1} \stackrel{\text{Lemma}}{\leq} \sum \frac{2 + 9x}{25} = \frac{1}{25} (3 \times 2 + 9(x + y + z)) \stackrel{x+y+z=1}{=} \frac{1}{25} (6 + 9) = \frac{15}{25} = \frac{3}{5}$$

Equality holds for an equilateral triangle.