ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$6 \leq \sum \frac{r_a + r}{r_a - r} \leq \frac{3R}{r}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\frac{r_a + r}{r_a - r} = \frac{r\left(\frac{s}{s-a} + 1\right)}{r\left(\frac{s}{s-a} - 1\right)} = \frac{2s - a}{a} = \frac{b + c}{a}$$

$$\sum \frac{r_a + r}{r_a - r} = \sum \frac{b + c}{a} = \sum \left(\frac{b}{a} + \frac{a}{b}\right) \overset{AM-GM}{\geq} 2 + 2 + 2 = 6$$

$$\sum \frac{r_a + r}{r_a - r} = \sum \frac{b + c}{a} = \sum \left(\frac{b}{a} + \frac{a}{b}\right) \overset{Bandila}{\leq} \sum \frac{R}{r} = \frac{3R}{r}$$

Equality holds for an equilateral triangle.