ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$r_a^2+r_b^2+r_c^2\ge m_a^2+m_b^2+m_c^2+2(R^2-4r^2)$$

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Solution by Tapas Das-India

$$r_a^2+r_b^2+r_c^2=\left(\sum r_a\right)^2-2\sum r_ar_b=2(4R+r)^2-2s^2,\ \sum m_a^2=\frac{3}{2}(s^2-r^2-4Rr)$$

$$We\ \ need\ to\ show: r_a^2+r_b^2+r_c^2\ge m_a^2+m_b^2+m_c^2+2(R^2-4r^2)$$

$$(4R+r)^2-2s^2\ge\frac{3}{2}(s^2-r^2-4Rr)+2(R^2-4r^2)$$

$$2(4R+r)^2-4s^2\ge 3(s^2-r^2-4Rr)+4(R^2-4r^2)$$

$$2(16R^2+8Rr+r^2)-4s^2\ge 3(s^2-r^2-4Rr)+4(R^2-4r^2)$$

$$28R^2+28Rr+21r^2\ge 7s^2$$

$$7(4R^2+4Rr+3r^2)\ge 7s^2\ true\ (Gerretsen)$$

Equality holds for an equilateral triangle.