ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\cot^2\frac{A}{2}+\cot^2\frac{B}{2}+\cot^2\frac{C}{2}\ge 3\left(\tan\frac{A}{2}+\tan\frac{B}{2}+\tan\frac{C}{2}\right)^2$$

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Solution by Tapas Das-India

$$We\ \ know\ that\ s^2\ge 3r(4R+r)$$

$$\cot^2\frac{A}{2}+\cot^2\frac{B}{2}+\cot^2\frac{C}{2}=\sum\cot^2\frac{A}{2}=\sum\frac{s^2}{r_a^2}=$$

$$=s^2\left(\left(\sum\frac{1}{r_a}\right)^2-2\sum\frac{1}{r_ar_b}\right)=s^2\left(\frac{1}{r^2}-\frac{2(4R+r)}{s^2r}\right)=$$

$$=\frac{s^2}{r^2}-\frac{2(4R+r)}{r}\ge\frac{3r(4R+r)}{r^2}-\frac{2(4R+r)}{r}=\frac{4R+r}{r}\ (1)$$

$$3\left(\tan\frac{A}{2}+\tan\frac{B}{2}+\tan\frac{C}{2}\right)^2=3\left(\frac{4R+r}{s}\right)^2=\frac{3(4R+r)^2}{s^2}\le$$

$$\le\frac{3(4R+r)^2}{3r(4R+r)}=\frac{4R+r}{r}\overset{(1)}{\le}\cot^2\frac{A}{2}+\cot^2\frac{B}{2}+\cot^2\frac{C}{2}$$

Equality holds for an equilateral triangle.