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In ΔABC the following relationship holds:

$$\left(\frac{h_a}{w_a}-\sin\frac{A}{2}\right)\left(\frac{h_b}{w_b}-\sin\frac{B}{2}\right)\left(\frac{h_c}{w_c}-\sin\frac{C}{2}\right)\le\frac{r}{4R}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\left(\frac{h_a}{w_a}-\sin\frac{A}{2}\right)=\frac{bc}{2R}\cdot\frac{b+c}{2bc\cos\frac{A}{2}}-\sin\frac{A}{2}=\left(\frac{1}{4R}\cdot\frac{2R(\sin B+\sin C)}{\cos\frac{A}{2}}-\sin\frac{A}{2}\right)=$$

$$=\frac{\sin\frac{B+C}{2}+\cos\frac{B-C}{2}}{\cos\frac{A}{2}}-\sin\frac{A}{2}\overset{A+B+C=\pi}{=}\cos\frac{B-C}{2}-\cos\frac{B+C}{2}=2\sin\frac{B}{2}\sin\frac{C}{2}\ (1)$$

$$\left(\frac{h_a}{w_a}-\sin\frac{A}{2}\right)\left(\frac{h_b}{w_b}-\sin\frac{B}{2}\right)\left(\frac{h_c}{w_c}-\sin\frac{C}{2}\right)\overset{(1)}{=}8\sin^2\frac{A}{2}.\sin^2\frac{B}{2}.\sin^2\frac{C}{2}=$$

$$=8\cdot\frac{r^2}{16R^2}=\frac{1}{2}\cdot\frac{r}{R}\cdot\frac{r}{R}\overset{Euler}{\le}\frac{r}{4R}$$

Equality holds for an equilateral triangle