

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\left(\tan \frac{A}{2}\right)^{\tan \frac{A}{2}} + \left(\tan \frac{B}{2}\right)^{\tan \frac{B}{2}} + \left(\tan \frac{C}{2}\right)^{\tan \frac{C}{2}} \geq 3 \left(\frac{\sqrt{3}}{3}\right)^{\frac{\sqrt{3}}{3}}$$

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Solution by Tapas Das-India

$$\sum \tan \frac{A}{2} = \frac{4R + r}{s} \stackrel{\text{Doucet}}{\geq} \sqrt{3} \quad (1)$$

Let $f(x) = x^x = e^{x \log x}$, ($\forall x > 0$), $f'(x) = e^{x \log x}(1 + \log x)$,
 $f''(x) = e^{x \log x} \frac{1}{x} + e^{x \log x}(1 + \log x)^2 > 0$

f is convex, using Jensen we get :

$$f\left(\tan \frac{A}{2}\right) + f\left(\tan \frac{B}{2}\right) + f\left(\tan \frac{C}{2}\right) \geq 3 f\left(\frac{\left(\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2}\right)}{3}\right) \stackrel{(1)}{\geq} 3 f\left(\frac{\sqrt{3}}{3}\right)$$

$$\left(\tan \frac{A}{2}\right)^{\tan \frac{A}{2}} + \left(\tan \frac{B}{2}\right)^{\tan \frac{B}{2}} + \left(\tan \frac{C}{2}\right)^{\tan \frac{C}{2}} \geq 3 \left(\frac{\sqrt{3}}{3}\right)^{\frac{\sqrt{3}}{3}}$$

Equality holds for an equilateral triangle.