ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1+\cos A}{\sin A}+\frac{1+\cos B}{\sin B}+\frac{1+\cos C}{\sin C}\ge 3\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{1+\cos A}{\sin A}+\frac{1+\cos B}{\sin B}+\frac{1+\cos C}{\sin C}=\sum_{cyc}\frac{1+\cos A}{\sin A}=$$

$$=\sum_{cyc}\frac{1+\cos\left(2\cdot\frac{A}{2}\right)}{\sin\left(2\cdot\frac{A}{2}\right)}=\sum_{cyc}\frac{1+2\cos^2\frac{A}{2}-1}{2\sin\frac{A}{2}\cos\frac{B}{2}}=$$

$$=\sum_{cyc}\cot\frac{A}{2}\overset{JENSEN}{\geq}3\cot\left(\frac{A+B+C}{6}\right)=3\cot\frac{\pi}{6}=3\sqrt{3}$$

Equality holds for $A=B=C$.