ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(1-\cos A)(1-\cos B)(1-\cos C) \geq \cos A \cos B \cos C$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$(1-\cos A)(1-\cos B)(1-\cos C) \geq \cos A \cos B \cos C$$

$$1-\sum \cos A+\sum \cos A \cos B-\prod \cos A \geq \prod \cos A$$

$$1-\sum \cos A+\sum \cos A \cos B-2\prod \cos A \geq 0$$

$$1-\sum \cos A+\sum \cos A \cos B-2\prod \cos A=$$

$$=1-\left(1+\frac{r}{R}\right)+\frac{s^2+r^2-4R^2}{4R^2}-2.\frac{s^2-(2R+r)^2}{4R^2}=$$

$$=\frac{1}{4R^2}\left(2(2R+r)^2-4R^2+r^2-4Rr-s^2\right)=$$

$$=\frac{1}{4R^2}\left(4R^2+4Rr+3r^2-s^2\right) \overset{GERRETSEN}{\geq} \frac{1}{4R^2}\left(s^2-s^2\right)=0$$

Equality holds for an equilateral triangle.