ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a}{\sin\frac{A}{2}}+\frac{m_b}{\sin\frac{B}{2}}+\frac{m_c}{\sin\frac{C}{2}}\ge\frac{a^2+b^2+c^2}{2r}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$Using\ the\ known\ inequalities\ m_a\ge\frac{b+c}{2}\cos\frac{A}{2}\ and$$

$$\frac{m_a}{\sin\frac{A}{2}}\ge\frac{\frac{b+c}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}=\frac{b+c}{2}\cot\frac{A}{2}=\frac{b+c}{2}.\frac{s}{r_a}=\frac{s(2s-a)}{2r_a}=\frac{2s^2}{2r_a}-\frac{as}{2r_a}=$$

$$=\frac{2s^2}{2r_a}-\frac{a(s-a)}{2r}\ (1)$$

$$\frac{m_a}{\sin\frac{A}{2}}+\frac{m_b}{\sin\frac{B}{2}}+\frac{m_c}{\sin\frac{C}{2}}=\sum\frac{m_a}{\sin\frac{A}{2}}\overset{(1)}{\ge}\sum\frac{2s^2}{2r_a}-\sum\frac{a(s-a)}{2r}=$$

$$=\frac{2s^2}{2r}-\frac{2s^2-(a^2+b^2+c^2)}{2r}=\frac{a^2+b^2+c^2}{2r}$$

$$Equality\ holds\ for\ A=B=C$$