ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$h_a + h_b + h_c - 9r \le 2(R - 2r)$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$h_a + h_b + h_c = \frac{ab + bc + ca}{2R} = \frac{s^2 + r^2 + 4Rr}{2R} \overset{Gerretsen}{\le}$$

$$\le \frac{4R^2 + 4Rr + 3r^2 + r^2 + 4Rr}{2R} = \frac{4R^2 + 8Rr + 4r^2}{2R}$$

$$We\ need\ \ to\ show:$$

$$h_a + h_b + h_c - 9r \le 2(R - 2r)$$

$$\frac{4R^2 + 8Rr + 4r^2}{2R} - 9r \le 2(R - 2r)$$

$$4R^2 - 10Rr + 4r^2 \le 4R^2 - 8Rr$$

$$2Rr \ge 4r^2\ or\ R \ge 2r\ (True\ Euler)$$

$$Equality\ holds\ for\ an\ equilateral\ triangle$$