ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{h_a}{a}+\frac{h_b}{b}+\frac{h_c}{c}\geq\frac{3\sqrt{3}r}{R}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\sum_{cyc}\frac{h_a}{a}=\sum_{cyc}\frac{2F}{a^2}=2F\sum_{cyc}\frac{1}{a^2}=2F\sum_{cyc}\frac{1^3}{a^2}\overset{RADON}{\geq}2F\cdot\frac{(1+1+1)^3}{(a+b+c)^2}=$$

$$=2F\cdot\frac{27}{4s^2}=\frac{27rs}{2s^2}=\frac{27r}{2s}\overset{MITRINOVIC}{\geq}\frac{27r}{2\cdot\frac{3\sqrt{3}}{2}R}=\frac{9r}{\sqrt{3}\cdot R}=\frac{3\sqrt{3}r}{R}$$

Equality holds for $a=b=c$.