

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{\sin A}{\cot A} + \frac{\sin B}{\cot B} + \frac{\sin C}{\cot C} \geq \frac{9}{2}$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \sum_{cyc} \frac{\sin A}{\cot A} &= \sum_{cyc} \frac{\sin^2 A}{\cos A} = \sum_{cyc} \frac{1 - \cos^2 A}{\cos A} = \sum_{cyc} \frac{1}{\cos A} - \sum_{cyc} \cos A = \\ &= \sum_{cyc} \sec A - \left(1 + \frac{r}{R}\right) \stackrel{JENSEN}{\geq} 3 \sec\left(\frac{A+B+C}{3}\right) - 1 - \frac{r}{R} = \\ &= 3 \sec \frac{\pi}{3} - 1 - \frac{r}{R} \stackrel{EULER}{\geq} 3 \cdot 2 - 1 - \frac{1}{2} = \frac{9}{2} \end{aligned}$$

Equality holds for $A = B = C$.