ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r}{R}+\frac{(a-b)^2+(b-c)^2+(c-a)^2}{16R^2}\le\frac{1}{2}$$

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Solution by Daniel Sitaru-Romania

$$\frac{r}{R}+\frac{(a-b)^2+(b-c)^2+(c-a)^2}{16R^2}=\frac{r}{R}+\frac{2(a^2+b^2+c^2-ab-bc-ca)}{16R^2}=$$

$$=\frac{r}{R}+\frac{(a^2+b^2+c^2)-(ab+bc+ca)}{8R^2}=$$

$$=\frac{r}{R}+\frac{(2s^2-2r^2-8Rr)-(s^2+r^2+4Rr)}{8R^2}=$$

$$=\frac{r}{R}+\frac{s^2-12Rr-3r^2}{8R^2}\overset{GERRETSEN}{\le}\frac{r}{R}+\frac{4R^2+4Rr+3r^2-12Rr-3r^2}{8R^2}=$$

$$=\frac{r}{R}+\frac{4R^2-8Rr}{8R^2}=\frac{r}{R}+\frac{1}{2}-\frac{r}{R}=\frac{1}{2}$$

Equality holds for $a=b=c$.